OCR
RECOGNISING ACHIEVEMENT

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS 4721

Core Mathematics 1

Monday **16 JANUARY 2006** Morning 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
List of Formulae (MF1)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- **You are not permitted to use a calculator in this paper.**

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

WARNING

You are not allowed to use a calculator in this paper.

This question paper consists of 3 printed pages and 1 blank page.

[Turn over

1 Solve the equations

(i) $x^{\frac{1}{3}} = 2$, [1]

(ii) $10^t = 1$, [1]

(iii) $\left(y^{-2}\right)^2 = \frac{1}{81}$. [2]

2 (i) Simplify $(3x+1)^2 - 2(2x-3)^2$. [3]

(ii) Find the coefficient of x^3 in the expansion of

$$(2x^3 - 3x^2 + 4x - 3)(x^2 - 2x + 1).$$ [2]

3 Given that $y = 3x^5 - \sqrt{x} + 15$, find

(i) $\frac{dy}{dx}$, [3]

(ii) $\frac{d^2y}{dx^2}$. [2]

4 (i) Sketch the curve $y = \frac{1}{x^2}$. [2]

(ii) Hence sketch the curve $y = \frac{1}{(x-3)^2}$. [2]

(iii) Describe fully a transformation that transforms the curve $y = \frac{1}{x^2}$ to the curve $y = \frac{2}{x^2}$. [3]

5 (i) Express $x^2 + 3x$ in the form $(x+a)^2 + b$. [2]

(ii) Express $y^2 - 4y - \frac{11}{4}$ in the form $(y+p)^2 + q$. [2]

A circle has equation $x^2 + y^2 + 3x - 4y - \frac{11}{4} = 0$.

(iii) Write down the coordinates of the centre of the circle. [1]

(iv) Find the radius of the circle. [2]

6 (i) Find the coordinates of the stationary points on the curve $y = x^3 - 3x^2 + 4$. [6]

(ii) Determine whether each stationary point is a maximum point or a minimum point. [3]

(iii) For what values of x does $x^3 - 3x^2 + 4$ increase as x increases? [2]

7 **(i)** Solve the equation $x^2 - 8x + 11 = 0$, giving your answers in simplified surd form. [4]

(ii) Hence sketch the curve $y = x^2 - 8x + 11$, labelling the points where the curve crosses the axes. [3]

(iii) Solve the equation $y - 8y^{\frac{1}{2}} + 11 = 0$, giving your answers in the form $p \pm q\sqrt{5}$. [4]

8 **(i)** Given that $y = x^2 - 5x + 15$ and $5x - y = 10$, show that $x^2 - 10x + 25 = 0$. [2]

(ii) Find the discriminant of $x^2 - 10x + 25$. [1]

(iii) What can you deduce from the answer to part **(ii)** about the line $5x - y = 10$ and the curve $y = x^2 - 5x + 15$? [1]

(iv) Solve the simultaneous equations

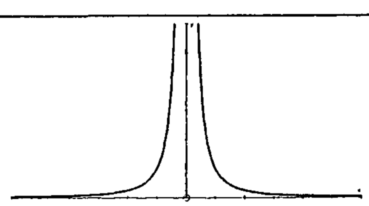
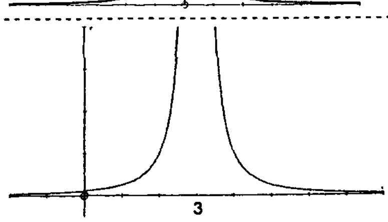
$$y = x^2 - 5x + 15 \quad \text{and} \quad 5x - y = 10.$$ [3]

(v) Hence, or otherwise, find the equation of the normal to the curve $y = x^2 - 5x + 15$ at the point $(5, 15)$, giving your answer in the form $ax + by = c$, where a, b and c are integers. [4]

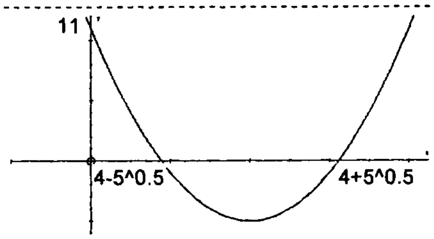
9 The points A, B and C have coordinates $(5, 1)$, $(p, 7)$ and $(8, 2)$ respectively.

(i) Given that the distance between points A and B is twice the distance between points A and C, calculate the possible values of p. [7]

(ii) Given also that the line passing through A and B has equation $y = 3x - 14$, find the coordinates of the mid-point of AB. [4]

1	(i)	$x^{\frac{1}{3}} = 2$ $x = 8$	B1	1	(allow embedded values throughout question 1) 8
	(ii)	$10^t = 1$ $t = 0$	B1	1	0
	(iii)	$\left(y^{-2}\right)^2 = \frac{1}{81}$ $y^{-4} = \frac{1}{81}$ $y = \pm 3$	B1 B1	2	$y = 3$ $y = -3$
2	(i)	$(3x+1)^2 - 2(2x-3)^2$ $= (9x^2+6x+1) - 2(4x^2-12x+9)$ $= x^2+30x-17$	M1 A1 A1	3	Square to get at least one 3 or 4 term quadratic $9x^2+6x+1$ or $4x^2-12x+9$ soi $x^2+30x-17$
	(ii)	$2x^3+6x^3+4x^3 = 12x^3$ 12	B1 B1	2	2 of $2x^3$, $6x^3$, $4x^3$ soi **N.B. www for these terms,** must be positive 12 or $12x^3$
3	(i)	$\frac{dy}{dx} = 15x^4 - \frac{1}{2}x^{-\frac{1}{2}}$	B1 B1 B1	3	$15x^4$ $kx^{-\frac{1}{2}}$ $cx^4 - \frac{1}{2}x^{-\frac{1}{2}}$ only
	(ii)	$\frac{d^2y}{dx^2} = 60x^3 + \frac{1}{4}x^{-\frac{3}{2}}$	M1 A1	2	Attempt to differentiate their 2 term $\frac{dy}{dx}$ and get one correctly differentiated term $60x^3 + \frac{1}{4}x^{-\frac{3}{2}}$
4	(i)		B1 B1	2	Correct curve in one quadrant Completely correct
	(ii)		M1 A1√	2	Translate (i) horizontally Translates all of their (i) $\begin{pmatrix}3\\0\end{pmatrix}$ 3 must be labelled or stated
	(iii)	(One-way) stretch, sf 2, parallel to the y-axis	B1 B1 B1	3	Stretch (Scale) factor 2 Parallel to y-axis o.e. **SR** Stretch B1 Sf $\sqrt{2}$ parallel to x-axis B2

5	(i)	$x^2+3x=\left(x+\frac{3}{2}\right)^2-\frac{9}{4}$	B1 B1	2	$a=\frac{3}{2}$ $b=-\frac{9}{4}$ o.e.
	(ii)	$y^2-4y-\frac{11}{4}=(y-2)^2-\frac{27}{4}$	B1 B1	2	$p=-2$ $q=-\frac{27}{4}$ o.e.
	(iii)	Centre $\left(-\frac{3}{2},2\right)$	B1√	1	$\left(-\frac{3}{2},2\right)$ N.B. If question is restarted in this part, ft from part (iii) working only
	(iv)	Radius $=\sqrt{\frac{27}{4}+\frac{9}{4}}$ $=\sqrt{9}$ $=3$	M1 A1	2	$\sqrt{-their'b'-their'q'}$ or use $\sqrt{(f^2+g^2-c)}$ 3 (±3 scores A0)
6	(i)	$y=x^3-3x^2+4$ $\frac{dy}{dx}=3x^2-6x$ $3x^2-6x=0$ $3x(x-2)=0$ $x=0 \quad x=2$ $y=4 \quad y=0$	B1 B1 M1 M1 A1 A1√	6	$3x^2-6x$ 1 term correct Completely correct $\frac{dy}{dx}=0$ Correct method to solve quadratic $x=0, 2$ $y=4, 0$ **SR** one correct (x,y) pair **www** **B1**
	(ii)	$\frac{d^2y}{dx^2}=6x-6$ $x=0 \quad y''=-6$ −ve max $x=2 \quad y''=6$ +ve min	M1 B1 B1	3	Correct method to find nature of stationary points (can be a sketch) $x=0$ max $x=2$ min (N.B. If no method shown but both min and max correctly stated, award all 3 marks)
	(iii)	Increasing $x<0 \quad x>2$	M1 A1	2	Any inequality (or inequalities) involving both their x values from part (i) Allow $x\le 0 \quad x\ge 2$

7	(i)	$x = \dfrac{8 \pm \sqrt{64-44}}{2}$	M1		Correct use of formula
		$= \dfrac{8 \pm \sqrt{20}}{2}$	A1		$\dfrac{8 \pm \sqrt{20}}{2}$ aef
			B1		$\sqrt{20} = 2\sqrt{5}$ soi
		$= 4 \pm \sqrt{5}$	A1	4	$4 \pm \sqrt{5}$
					<u>Alternative method</u> $(x-4)^2 - 16 + 11 = 0$ M1 $(x-4)^2 = 5$ A1 $x = 4 + \sqrt{5}$ A1 or $4 - \sqrt{5}$ A1
	(ii)		B1 B1√ B1	3	+ve parabola Root(s) in correct places Completely correct curve with roots and (0, 11) labelled or referenced
	(iii)	$y = x^2 = \left(4 \pm \sqrt{5}\right)^2$	M1 M1		$y = x^2$ soi Attempt to square at least one answer from part (i)
		$= 16 + 5 \pm 8\sqrt{5}$	A1√		Correct evaluation of $(a + b\sqrt{c})^2$ $(a,b,c \neq 0)$
		$= 21 \pm 8\sqrt{5}$	A1	4	$21 \pm 8\sqrt{5}$

8	(i)	$y = x^2 - 5x + 15$ $y = 5x - 10$ $x^2 - 5x + 15 = 5x - 10$ $x^2 - 10x + 25 = 0$	M1 A1	 2	Attempt to eliminate y $x^2 - 10x + 25 = 0$ **AG** Obtained with no wrong working seen
	(ii)	$b^2 - 4ac = 100 - 100$ $= 0$	B1	1	0 Do not allow $\sqrt{(b^2 - 4ac)}$
	(iii)	Line is a tangent to the curve	B1√	1	Tangent or 'touches' N.B. Strict ft from their discriminant
	(iv)	$x^2 - 10x + 25 = 0$ $(x-5)^2 = 0$ $x = 5 \quad y = 15$	M1 A1 A1	 3	Correct method to solve 3 term quadratic $x = 5$ $y = 15$
	(v)	Gradient of tangent = 5 Gradient of normal = $-\frac{1}{5}$ $y - 15 = -\frac{1}{5}(x-5)$ $x + 5y = 80$	B1 B1√ M1 A1	 4	Gradient of tangent = 5 Gradient of normal = $-\frac{1}{5}$ Correct equation of straight line, any gradient, passing through (5, 15) $x + 5y = 80$

9	(i)	Length AC = $\sqrt{(8-5)^2+(2-1)^2}$ $=\sqrt{3^2+1^2}$ $=\sqrt{10}$	M1 A1		Uses $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ $\sqrt{10}$ ($\pm\sqrt{10}$ scores A0)
		Length AB = $\sqrt{(p-5)^2+(7-1)^2}$ $=\sqrt{(p-5)^2+36}$	A1		$\sqrt{(p-5)^2+(7-1)^2}$
		$\sqrt{(p-5)^2+36}=2\sqrt{10}$ $p^2-10p+25+36=40$ $p^2-10p+21=0$ $(p-7)(p-3)=0$ $p=7,3$	M1 M1 A1 A1	7	AB = 2AC (with algebraic expression) used Obtains 3 term quadratic = 0 suitable for solving <u>or</u> $(p-5)^2=4$_ $p=7$ $p=3$ **SR <u>If no working seen</u>, and one correct value found, award B2 in place of the final 4 marks in part (i)**
	(ii)	$7=3x-14$ $x=7$ (5, 1) (7, 7) Mid-point (6, 4)	M1 A1 M1 A1√	4	Correct method to find x $x=7$ Use $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ (6, 4) or correct midpoint for their AB <u>Alternative method</u> y coordinate of midpoint $=4$ M1 A1 sub 4 into equation of line M1 obtains $x=6$ A1